ChE-304 Midterm Exam

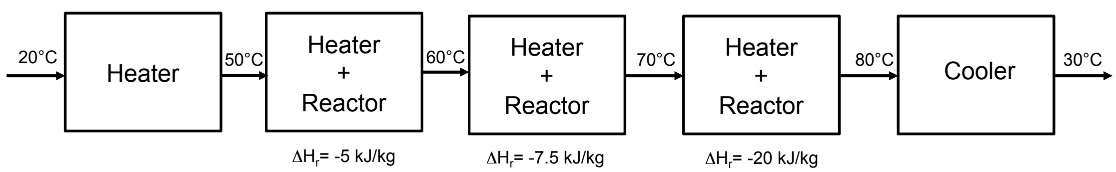
April 3rd 2016. Time: 3 hours (15:15-18:15).

Each problem is worth 20 points.

Problem 1

Stream A (a liquid during the entire process) goes through the process system below. Throughout the process, this stream has a constant Cp ($Cp_{Stream\ A}$=1 kJ/kg) that stays the same at all temperatures and is unaffected by any reactions that are happening.

Stream A goes through several reactors during which **exothermic** reactions occur. For each reactor a certain heat of reaction is released that is proportional to conversion and conversion is linearly dependent on the increase in temperature. Basically, this means that the heat released in each reactor is linearly dependent on the temperature increase.



Can you sketch the hot and cold composite curves (without any temperature corrections) and indicate what you think the minimum approach temperature should be on your drawing?

We are not looking for an exact drawing to scale but a drawing that reveals the general trends of the curves (e.g. the relative steepness of the slopes should be correct, etc.). Include at what temperatures events are happening. If you are not confident that your drawing made something clear, annotate it (e.g. values of the slope, etc.).

Solution:

Streams

1) 20°C – 50°C Cp = 1 kJ/ (kg K) Q = 1 kg (50 – 20) 1 = 30 kJ → Cold stream
2) 50°C – 60°C Cp = 1 kJ/ (kg K) Q = 1 kg (60 – 50) 1 – 5 = 5 kJ → Cold stream
3) 60°C – 70°C Cp = 1 kJ/ (kg K) Q = 1 kg (70 – 60) 1 – 7.5 = 2.5 kJ → Cold stream
4) 70°C – 80°C Cp = 1 kJ/ (kg K) Q = 1 kg (80 – 70) 1 – 20 = -10 kJ → Hot stream
5) 80°C – 30°C Cp = 1 kJ/ (kg K) Q = 1 kg (30 – 80) 1 = -50 kJ → Hot stream

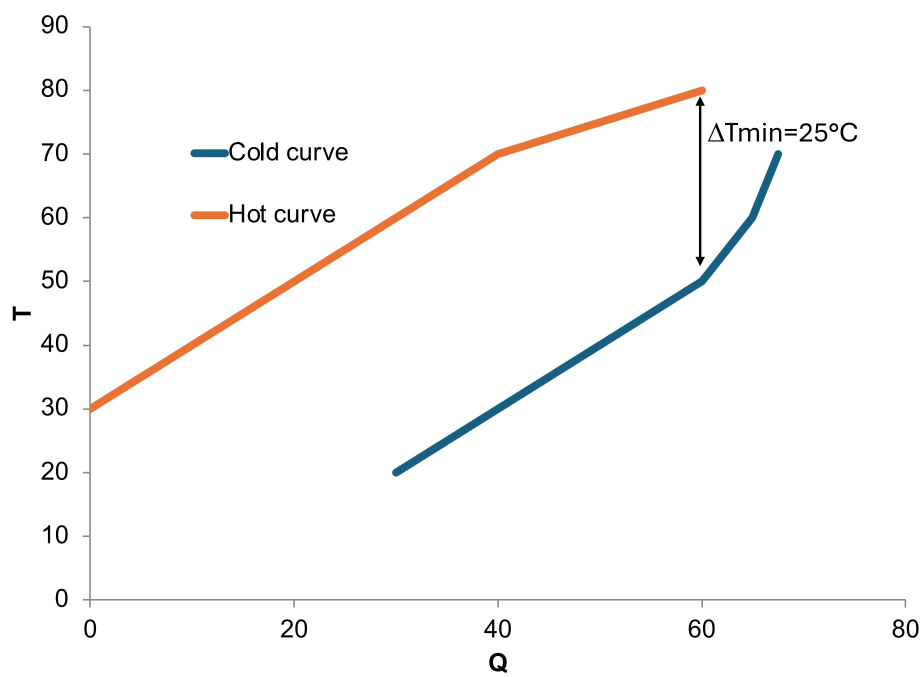
Cold composite curves:

1) 20°C – 50°C Cp = 1 kJ/ (kg K) Q = 1 kg (50 – 20) 1 = 30 kJ
2) 50°C – 60°C Cp = 1 kJ/ (kg K) Q = 1 kg (60 – 50) 1 – 5 = 5 kJ
3) 60°C – 70°C Cp = 1 kJ/ (kg K) Q = 1 kg (70 – 60) 1 – 7.5 = 2.5 kJ

Hot Composite curve:

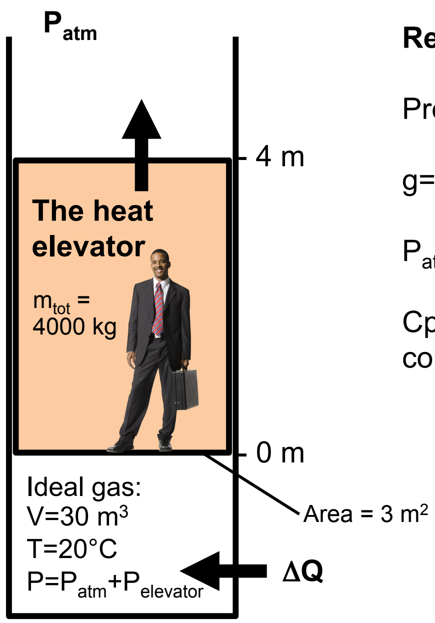
1) 80°C – 70°C Cp = 1 kJ/ (kg K) Q = 1 kg (80 – 70) 1 + 1 kg (70 – 80) 1 – 20 = -20 kJ
2) 70°C – 30°C Cp = 1 kJ/ (kg K) Q = 1 kg (30 – 70) 1 = -40 kJ

We can start with zero for the hot composite curve



Problem 2

We have built a heat elevator that is able to raise itself by adding heat to a completely closed volume of ideal gas that is under the elevator (i.e. the elevator acts like a piston with an area of 3 m^2). At the ground floor the elevator is at 0 m. How much heat must be added to the gas for the elevator to rise to the first floor, which is at 4 m?



Recall:

Pressure = Force/Area=(mass*acceleration)/Area

g= 9.8 m/s^2

P_{atm}=101'325 Pa = 101'325 kg m^{-1} s^{-2}

$Cp_{ideal\ gas}$=30 J / (mol K) (which we assume to be constant with T°)

Solution

The total pressure under the elevator is:

$$P = P_{atm} + P_{elevator} = 101325 + \frac{F}{Area} = 101325 + \frac{m * g}{Area} = 101325 + 4000 * \frac{9.8}{3}$$
$$= 114.4\ kPa$$

Because it's an ideal gas and we know the pressure stays constant and we know the final initial and final volume, we can calculate the final temperature.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ @ cst } T \rightarrow T_2 = T_1 \frac{V_2}{V_1} = 293K * \frac{42m^3}{30m^3} = 410\ K$$

The total amount of work is equal to the work necessary to push up the elevator and the work to push the atmosphere out of the way:

$$W_{tot} = W_{piston} + W_{atm} = -m * g * h - P_{atm}\Delta V = -(P_{elevator} + P_{atm})\Delta V$$
$$= -114.4 * 12 = -1373\ kJ$$

Note that given how we calculated $P_{elevator}$ we can write that $m * g * h$ is equal to $P_{elevator}\Delta V$.

If we do a first law balance around the ideal gas, we have that:

$$\Delta U = Q + W_{tot} \rightarrow Q = \Delta U - W_{tot}$$

For an ideal gas: $\Delta U = C_V \Delta T = 20(410 - 293) = 2340\ J$/mol

$$Q = 2340 * n_{mol} + 1373000 = 2340 * \frac{30 * 114500}{8.314 * 293} + 1373000 = 4.67\ MJ$$

Problem 3

When you heat your house with fuel oil ("mazout") you produce gases in a continuous process in the burner that are around 1400 °C (1673 K). These gases are ultimately cooled to around 100°C in order to heat a home at a temperature that is generally constant at around 20°C (and no work is produced in the process). This is not efficient because a tremendous amount of work/exergy is wasted by not exploiting this temperature difference. Can you calculate the maximum amount of work that is lost as a fraction of the heat used to heat the house?

The gases can be considered ideal and they are always at a pressure of 1 atm (101325 Pa).

The Cp can be assumed to be independent of temperature and equal to:

Cp= 30 J/(mol K) (assumed to be constant with T°)

Solution:

We calculate the exergy of the gas going from 1400°C to 100°C

$$W_{ex,1\rightarrow 2} = -(H_2 - H_1) + T_0(S_2 - S_1)$$

With a constant Cp:

$$\Delta H = Cp\Delta T = -30 * 1300 = -39'000\,J$$

For a constant P path:

$$dQ_{rev} = dU - dW_{rev} = nC_V dT + nRdT = nC_P dT$$

$$\Delta S = \int \frac{dQ_{rev}}{T} = \int \frac{nC_P dT}{T} = nC_P \ln\left(\frac{T_2}{T_1}\right)$$

$$= 30 * \ln\left(\frac{373}{1673}\right) = -45\,J/K$$

$$W_{ex,1\rightarrow 2} = 39'000 - 293 * 45 = 25'815 \text{ J}$$

$$W_{lost} = W_{ex,1\rightarrow 2} - W_{produced} = W_{ex,1\rightarrow 2} = 25'815 \text{ J}$$

Fraction of work lost for the heat produced: $\frac{W_{lost}}{\Delta H} = 66\%$